

JEE MAIN 2026

Sample Paper – 6

Time Allowed: 3 hours

Maximum Marks: 300

General Instructions:

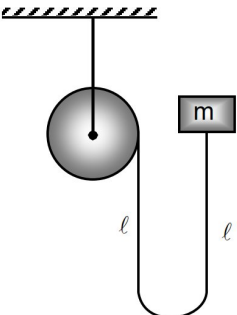
1. The test consists of total **75 questions**.
2. Each subject (**PCM**) has **25 questions**.
3. Each subject divided into two sections. **Section A** consists of 20 multiple-choice questions & **Section B** consists of 5 numerical value-type questions.
4. **Marking Scheme:**
 - **Section A (MCQs):** +4 marks for each correct answer, –1 mark for each incorrect answer, 0 marks for unattempted.
 - **Section B (Numerical):** +4 marks for each correct answer, 0 marks for incorrect or unattempted.
5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

SECTION – A

(Single Choice Answer Type)

This section contains **20 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

- A block of mass m is attached to one end of a light string which is wrapped on a disc of mass $2m$ and radius R which is free to rotate about the horizontal axis passing through its centre of mass. The total length of the slack portion of the string is ℓ . The block is released from rest as shown. The velocity of the block just after the string becomes taut is (no slipping between disc and string)



(A) $\sqrt{g\ell}$ (B) $\sqrt{\frac{g\ell}{2}}$
(C) $\sqrt{\frac{2g\ell}{3}}$ (D) $\sqrt{\frac{3g\ell}{2}}$
- A potential of 2 V is applied across the faces of a pure germanium plate of area $4 \times 10^{-4}\text{ m}^2$ and of thickness $2 \times 10^{-3}\text{ m}$. Concentration of carriers in germanium at room temperature is $1.0 \times 10^6\text{ m}^{-3}$, Mobility of electrons and holes are $0.7\text{ m}^2\text{V}^{-1}\text{s}^{-1}$ and $0.3\text{ m}^2\text{V}^{-1}\text{s}^{-1}$ respectively. The current produced in germanium plate at room temperature, is

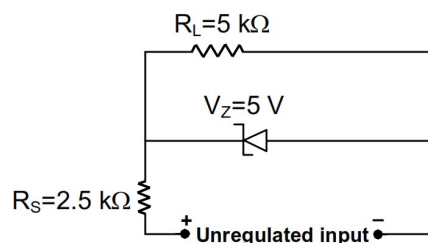
(A) $1.40 \times 10^{-10}\text{ A}$ (B) $1.40 \times 10^{-9}\text{ A}$
(C) $9.6 \times 10^{-10}\text{ A}$ (D) $6.4 \times 10^{-14}\text{ A}$
- The wavelength of the waves associated with a proton and a photon are the same. Therefore, the two have equal.

(A) mass (B) velocity
(C) momentum (D) kinetic energy
- One mole of an ideal diatomic gas undergoes a thermodynamic process, in which its molar heat capacity varies directly proportional to temperature as $C = \alpha T$, where α is a positive constant. Work done by the gas when it is heated from initial temperature T_0 to a final temperature $3T_0$ will be

(A) $4\alpha T_0^2$ (B) $(\alpha T_0 - R)\frac{3T_0}{2}$
(C) $(4\alpha T_0 - 5R)T_0$ (D) $(3\alpha T_0 - 5R)\frac{T_0}{2}$
- When a sample of atoms is irradiated by neutrons, radioactive atoms are produced at a constant rate R , which decay with decay constant λ . The number of radioactive atoms accumulated after an irradiation time t is given by

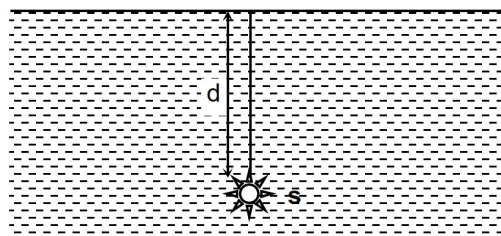
(A) $N(t) = Rte^{-\lambda t}$ (B) $N(t) = \frac{R}{\lambda}e^{-\lambda t}$
(C) $N(t) = \frac{R}{\lambda}(1 - e^{-\lambda t})$ (D) $N(t) = Rt(1 - e^{-\lambda t})$

6. In the DC voltage regulator circuit shown, the zener breakdown voltage $V_Z = 5\text{ V}$. If the unregulated input varies between 11 V to 15 V, maximum zener current (in mA) is
- (A) 1 (B) 2
(C) 3 (D) 4

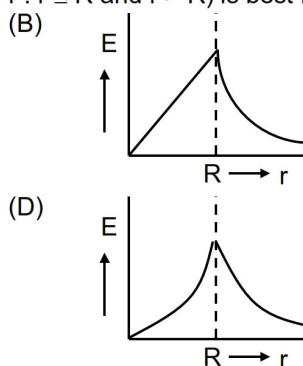
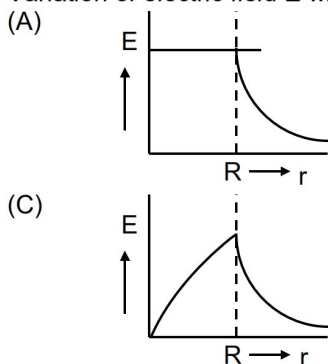


7. Radius of $^{32}_{32}\text{Ge}$ (germanium) nucleus is measured to be twice the radius of ^9_4Be nucleus. Number of neutrons in Ge are
- (A) 38 (B) 40
(C) 42 (D) 44
8. A SONAR system fixed in a submarine operates at a frequency 40 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km/h. The frequency of sound is reflected by submarine is (Speed of sound in water = 1400 m/s)
- (A) $\frac{300}{7}$ kHz (B) $\frac{600}{13}$ kHz
(C) $\frac{560}{13}$ kHz (D) 40 kHz

9. A point source S of light is placed at a depth d below the surface of water in a large and deep lake. Maximum fraction of light that escapes in space above directly from water (refractive index = μ) surface is given by



- (A) $\frac{1}{2} - \sqrt{\frac{\mu^2 - 1}{\mu}}$ (B) $\sqrt{\frac{\mu^2 - 1}{\mu}}$
(C) $\frac{1}{2} - \frac{\sqrt{\mu^2 - 1}}{2\mu}$ (D) $\frac{1}{2} - \frac{\sqrt{\mu^2 - 1}}{\mu^2}$
10. In a sphere of radius R , the volume charge density ρ varies as $\rho \propto r$ (r is distance from centre). Variation of electric field E with r (For all values of r : $r \leq R$ and $r > R$) is best represented by

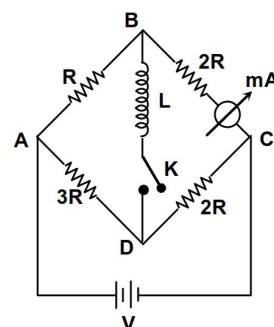


11. Binding energy per nucleon of Helium and deuterium nuclei are 7 MeV and 1.1 MeV, respectively. When two deuterium nuclei undergo fusion to form helium nucleus, energy released in the process is
 (A) 23.6 J (B) 37.76×10^{-13} MeV
 (C) 23.6 eV (D) 37.76×10^{-13} J

12. Current I flows through a long thin walled metallic cylinder of radius R with a thin longitudinal slit of width d_0 ($d_0 \ll R$) running parallel to the axis of the cylinder. The magnetic induction B produced at any point on the axis of the cylinder is approximately

- (A) $B = \text{zero}$ (B) $B = \frac{\mu_0 I}{2\pi R^2}$
 (C) $B = \frac{\mu_0 I d_0}{4\pi^2 R^2}$ (D) $B = \frac{\mu_0 I d_0}{2\pi R^2}$

13. The reading of the ammeter, used in the electrical network shown below, is 20 mA, a long time after the key K is closed. The reading of the same ammeter, immediately after the key was closed was
 (A) zero (B) 16 mA
 (C) 25 mA (D) $\frac{70}{3}$ mA

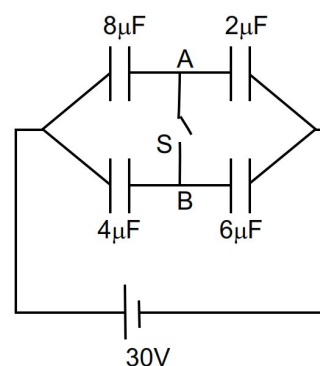


14. Consider the diffraction pattern due to a single slit. The first maximum for a certain monochromatic light coincides with the first minimum for red light of wavelength 660 nm. The wavelength of the monochromatic light is
 (A) 660 nm (B) 550 nm
 (C) 440 nm (D) 330 nm

15. The energy of the characteristic X-ray photon in Coolidge tube comes from
 (A) the kinetic energy of striking electron.
 (B) the kinetic energy of the free electrons of the target.
 (C) the kinetic energy of the ions of the target.
 (D) the electronic transition of the target atom.

16. A system of capacitors $C_1 = 8\mu\text{F}$, $C_2 = 2\mu\text{F}$, $C_3 = 4\mu\text{F}$ and $C_4 = 6\mu\text{F}$ connected across a battery of emf $E = 30$ V is shown in figure. The charge that will flow, through the switch S , when it is closed is

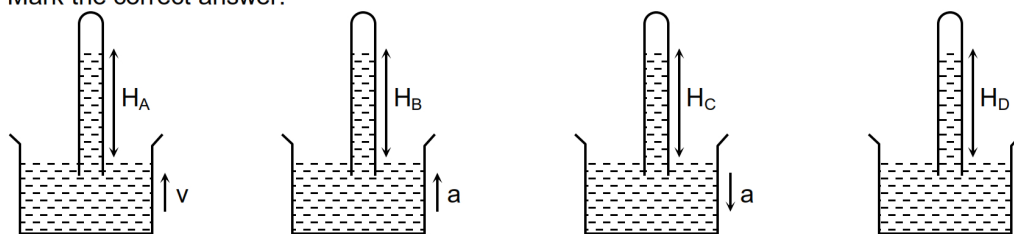
- (A) $60 \mu\text{C}$ A to B
 (B) $48 \mu\text{C}$ A to B
 (C) $24 \mu\text{C}$ B to A
 (D) $36 \mu\text{C}$ B to A



17. Four barometer A, B, C and D are shown in the figure. A is moving up with constant velocity, B is accelerated up, C is accelerated down and D is at rest. Atmospheric pressure is represented by 76 cm of mercury column.

(i) $H_A = 76$ cm, (ii) $H_B < 76$ cm, (iii) $H_C = 76$ cm, (iv) $H_D = 76$ cm

Mark the correct answer.



(A) (i), (ii), (iii), (iv)

(C) (i), (iii), (iv)

(B) (i), (ii), (iv)

(D) (ii), (iii), (iv)

18. A total charge Q is uniformly distributed over a non-conducting ring of radius r . There is a time varying magnetic field perpendicular to its plane and changing at the uniform rate of $\frac{dB}{dt}$.

The magnitude of torque experienced by the ring is

(A) $Qr^2 \left(\frac{dB}{dt} \right)$

(B) $Qr^3 \left(\frac{dB}{dt} \right)$

(C) $\frac{1}{2} Qr^2 \left(\frac{dB}{dt} \right)$

(D) $\frac{1}{2} Qr^3 \left(\frac{dB}{dt} \right)$

19. A circular loop of radius r is placed inside another circular loop of radius R ($R \gg r$). The loops are coplanar and concentric. The mutual inductance (M) of the system is proportional to

(A) $\frac{\mu_0 \pi r}{R}$

(B) $\frac{\mu_0 \pi r^2}{R}$

(C) $\frac{\mu_0 \pi R^2}{r}$

(D) $\frac{\mu_0 \pi r^2}{R^2}$

20. A sphere and cube have equal surface area are made of the same material. The two are heated to the same temperature and kept in identical surrounding. The ratio of their initial rates of cooling is

(A) $1 : 1$

(B) $\sqrt{\frac{\pi}{2}} : 1$

(C) $\sqrt{\frac{\pi}{3}} : 1$

(D) $\sqrt{\frac{\pi}{6}} : 1$

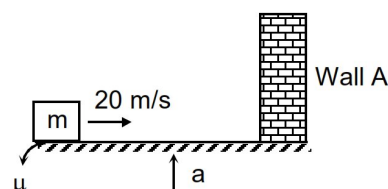
SECTION – B

(Numerical Answer Type)

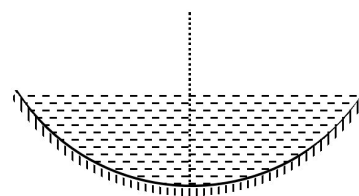
This section contains 5 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

21. In a young's double slit experiment distance between the slits is $d = 1 \text{ mm}$, Wavelength of light used is 600 nm and distance of screen from the plane of slits is $D = 1 \text{ m}$. The minimum distance between two points is $\frac{k}{10} \text{ mm}$ on the screen, where intensity is maximum. Find the value of k . (Assume both sources of equal power).

22. A block of mass m is projected from a point A with velocity 20 m/s on the rough surface having coefficient of friction μ to hit the wall B with velocity 10 m/s . With what velocity same mass m should be projected to hit the wall B with same velocity 10 m/s if the surface is moving now upward with an acceleration of $a = 4g$ as shown in the figure.



23. A sound source of constant frequency resonates with an open end organ pipe of length 30 cm and a close end organ pipe of length 23 cm (both of same diameter). Both pipes are sounding their first overtone. If velocity of sound is 340 ms^{-1} , then find the end correction in cm .
24. In an electromagnetic wave the amplitude of magnetic field is $3 \times 10^{-8} \text{ T}$. If the frequency of wave is 10^{12} Hz , then find the amplitude of associated electric field in S.I. unit.
25. A concave mirror when placed in air has a focal length $f = 20 \text{ cm}$. The mirror is now placed horizontally and filled with a thin layer of water having refractive index $\frac{4}{3}$. The object is placed at principal axis at a distance d from the mirror such that a real, inverted image coincides with the object. Find the value of d in cm .

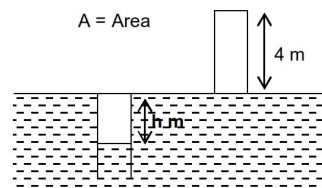


SECTION – A

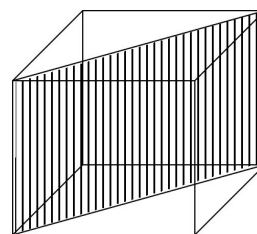
(Single Choice Answer Type)

This section contains **20 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

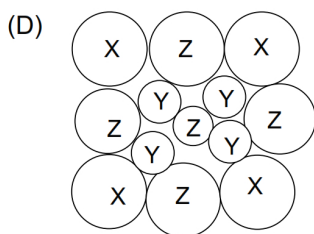
26. A 4 m long tube closed at one end is lowered vertically into water until the closed end is flushed with water surface (See figure). Calculate the water level height in the tube, h . [Barometric pressure = 1 atm = 10 m of hydrostatic water head; $T = 25^\circ\text{C}$, $d_{\text{H}_2\text{O}} = 1 \text{ g/mL}$. Neglect water vapour pressure]



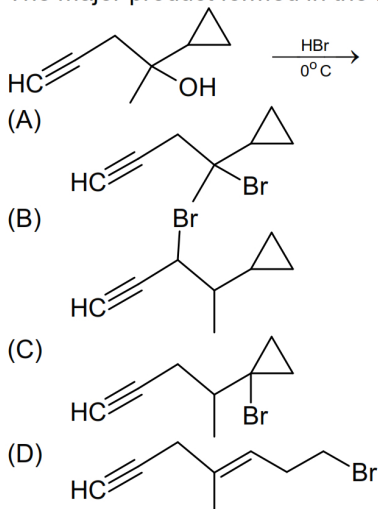
- (A) 1.71 m (B) 3.06 m
(C) 1.53 m (D) 0.855 m
27. The oxidation state of sulphur in the dithionous and dithionic acids, respectively are:
(A) +4, +6
(B) +4, +5
(C) +3, +5
(D) +3, +6
28. In a hypothetical solid, x atoms form CCP lattice. 'Y' atoms occupy all tetrahedral voids and 'Z' atoms occupy all octahedral voids. If a unit cell of crystal is cut by a plane shown in the figure, then the cross section of the plane looks like:



- (A)
- (B)
- (C)



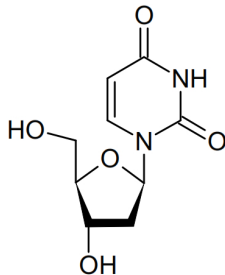
29. The major product formed in the following reaction is:



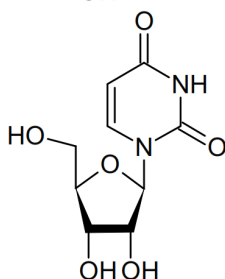
30. In the synthesis of polydimethyl siloxane, the chain forming, branching and terminating agent respectively, are
- (A) Me_2SiCl_2 , Me_3SiCl and MeSiCl_3
- (B) Me_2SiCl_2 , MeSiCl_3 and Me_3SiCl
- (C) MeSiCl_3 , Me_2SiCl_2 and Me_3SiCl
- (D) Me_2SiCl_2 , MeSiCl_3 and Me_4Si
31. Choose the correct statement(s) among the following
- (I) LiF is more soluble than LiClO_4 in water.
- (II) The standard reduction potential $[E^\circ]$ of Li is more negative than that of Na .
- (III) The heat of hydration of Li^+ (g) is greater than that of Na^+ (g).
- (A) I and II
- (B) I and III
- (C) II and III
- (D) III only
32. The two main pollutants which are released when fossil fuels are burnt are
- (A) O_3 and hydrocarbon
- (B) NO_2 and O_3
- (C) NO and NO_2
- (D) NO_2 and hydrocarbon

33. The role of BF_3 as an industrial polymerization catalyst is to generate
(A) Carbanion
(B) Carbocation
(C) Organic radical
(D) Carbon radical

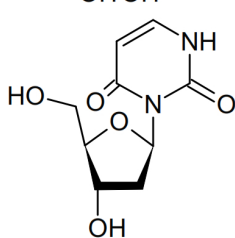
34. The correct structure of ribonucleoside uridine is:
(A)



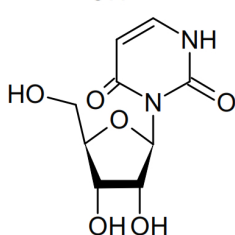
(B)



(C)

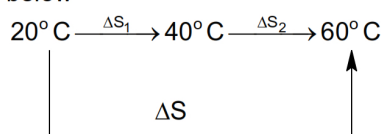


(D)



35. The number of degenerate spatial orbitals of a hydrogen like atom with principal quantum number $n = 5$ is
(A) 25
(B) 5
(C) 50
(D) 10

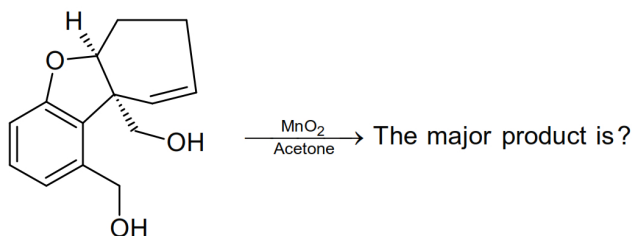
36. Consider the entropy changes in a system undergoing transformation, as depicted in the diagram below



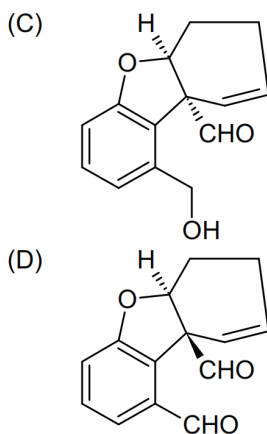
The correct statement among the following is

- (A) $\Delta S_1 = \Delta S_2$ and $\Delta S = \Delta S_1 + \Delta S_2$
 (B) $\Delta S_1 > \Delta S_2$ and $\Delta S \neq \Delta S_1 + \Delta S_2$
 (C) $\Delta S_1 < \Delta S_2$ and $\Delta S \neq \Delta S_1 + \Delta S_2$
 (D) $\Delta S_1 > \Delta S_2$ and $\Delta S = \Delta S_1 + \Delta S_2$
37. For the following complexes, the increasing order of magnetic moment (spin only value) is
- (a) $[\text{TiF}_6]^{-3}$ (b) $[\text{CrF}_6]^{-3}$ (c) $[\text{MnF}_6]^{-3}$ (d) $[\text{CoF}_6]^{-3}$
- (A) $d < a < b < c$
 (B) $c < a < d < b$
 (C) $b \approx a < d < c$
 (D) $a < b < c \approx d$
38. The stability of lyophobic colloids is a consequence of the
- (A) Electric double layer at the surface of the particle
 (B) van der Waal forces between the particles
 (C) Small size of particles
 (D) Shape of particles

39.



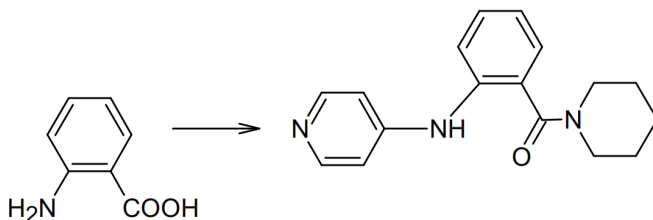
- (A)
- (B)



40. The rate constant of a second order reaction $A \longrightarrow B$ is k_2 . If the initial concentration of the reactant is x_0 and the concentration of the product at time t is a , then a linear function of t with the slope $k_2 x_0$ is :

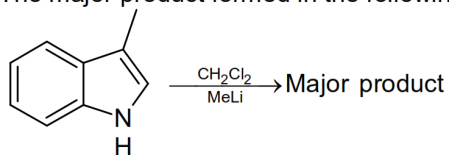
- (A) $\ln\left(\frac{a}{x_0 - a}\right)$
 (B) $\left(\frac{a}{x_0(x_0 - a)}\right)$
 (C) $\left(\frac{a}{x_0 - a}\right)$
 (D) $\ln\left(\frac{a}{x_0(x_0 - a)}\right)$

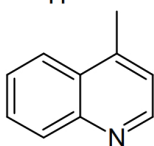
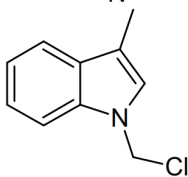
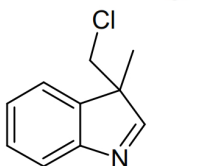
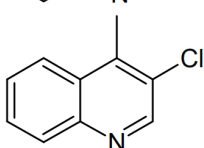
41. Correct sequence of reagent (i) to (iii) required for the conversion of A to B is:



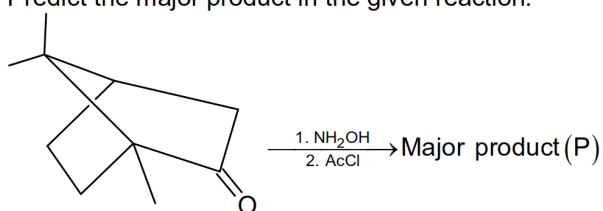
- (i) Thionyl chloride
 (ii) 4-chloropyridine
 (iii) Piperidine
 (A) i, ii and iii
 (B) i, iii and ii
 (C) ii, i and iii
 (D) iii, i and ii

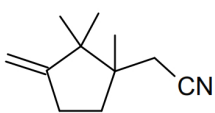
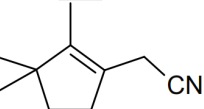
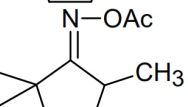
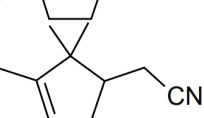
42. The major product formed in the following reaction is



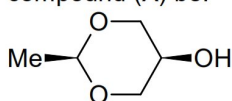
- (A) 
- (B) 
- (C) 
- (D) 

43. Predict the major product in the given reaction:



- (A) 
- (B) 
- (C) 
- (D) 

44. Among the structures given below, the one that corresponds to the most stable conformation of compound (X) be:



(X)

- (A)
- (B)
- (C)
- (D)

45. Allred – Rochow electronegativity of an element is:
- Directly proportional to the effective nuclear charge.
 - Directly proportional to the covalent radius.
 - Inversely proportional to the square of the covalent radius.
 - Directly proportional to the square of the effective nuclear charge.
- The correct answer is:
- (A) I and II
(B) I and III
(C) II and III
(D) I and IV

SECTION – B

(Numerical Answer Type)

This section contains 5 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

46. The degree of unsaturation in the product formed by heating mellitic acid ($C_6(COOH)_6$) is:
47. XH_2 (ion exchange resin) can replace Ca^{+2} ions in hard water as $XH_2 + Ca^{+2} \longrightarrow XCa + 2H^+$. If 1 L hard water after passing through XH_2 has a pH = 4 then hardness in parts per million of Ca^{+2} is
48. The theoretical efficiency of a hypothetical cell is about 84% which involves the following reaction
 $A(s) + B^{+2}(aq) \longrightarrow A^{+2}(aq) + B(s) \quad \Delta H = -285 \text{ kJ}$
 then, the standard EMF of the cell is:

49. An amount of 0.1 millimole of CdSO_4 is present in 10 mL acid solution of 0.08 M HCl. Now, H_2S is passed to precipitate all the Cd^{+2} ions. What would be the pH of solution after filtering off precipitate, boiling off H_2S and making the solution 100 mL by adding water?
50. The freezing point of an aqueous solution of KCN containing 0.2 mole/kg water was -0.8°C . On addition of 0.1 mole of $\text{Hg}(\text{CN})_2$ in the solution containing 1 Kg of water, the freezing point of the solution was -0.6°C . Assuming that the complex is formed according to the following equation:
$$\text{Hg}(\text{CN})_2 + m\text{CN}^- \longrightarrow \text{Hg}(\text{CN})_{m+2}^{-m}$$
 and $\text{Hg}(\text{CN})_2$ is the limiting reagent, the value of m is:



SECTION – A

(Single Choice Answer Type)

This section contains **20 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

51. Let $A = \{x : x^2 \leq x + 20,000 ; x \in \mathbb{N}\}$
 $B = \{x = 3k + 2 ; k \in \mathbb{N}\}$
 $C = \{x = 4k ; k \in \mathbb{N}\}$
 The sum of the elements of the set $A \cap (B - C)$ is
 (A) 2443 (B) 2447
 (C) 2450 (D) 2453
52. The number of integral values of a for which the equation $x^4 - (a + 2)x^3 + 2ax^2 + 4(a - 2)x - 16 = 0$ has at least two positive roots; $a \in [-10, 10]$ is/are
 (A) 5 (B) 6
 (C) 7 (D) 16
53. Let $(1 + x + x^2)^{30} = \sum_{r=0}^{60} a_r x^r$. If $\alpha a_{21} = \beta a_{20} + \gamma a_{19}$, ($\alpha, \beta, \gamma \in \mathbb{N}$) then the value of $\alpha + \beta + \gamma$ can be
 (A) 62 (B) 72
 (C) 82 (D) 74
54. $\int e^{\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)} \left(\frac{x^4 + x^2 - 1}{x^2}\right) dx = f(x) + c$, then $\left(f(\sqrt{2})\right)^4$ is
 (A) e^5 (B) $2e^5$
 (C) $4e^5$ (D) $\sqrt{2}e^5$
55. If $1(50)^{49} + 2(51)^1(50)^{48} + 3(51)^2(50)^{47} + \dots + (50)(51)^{49} = k(50)^{49}$ then the number of factor of k of the form $4m + 1$, ($m \in \mathbb{W}$) is/are
 (A) 3 (B) 4
 (C) 5 (D) 7
56. If the pair of perpendicular lines $4x^2 + by^2 + 2\cos\theta xy + 12x + 2\sin^2\theta y + c = 0$, $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$ intersect on x-axis then the area of triangle formed by the given pair of lines and $y = 2$ is
 (A) $\sqrt{65}$ (B) $\frac{\sqrt{65}}{2}$
 (C) $\frac{\sqrt{65}}{4}$ (D) $2\sqrt{65}$
57. Consider two circles
 $C_1 : (x - 1)^2 + (y - 4)^2 - 16 = 0$ and $C_2 : (x - 13)^2 + (y - 9)^2 - 81 = 0$
 If a circle of radius r touches x-axis and C_1 and C_2 externally, then r is equal to
 (A) 1.44 (B) 1.48
 (C) 1.52 (D) 1.6

58. A line $y = m(x - 4)$ meets x-axis at P and the parabola $x^2 = 32y$ at $Q(x_1, y_1)$. The tangent to the parabola at $Q(x_1, y_1)$ meets x-axis at $R(x_2, 0)$ $0 < x_2 < 6$. If the area of triangle PQR assumes a local maximum, then the value of m is
- (A) $\frac{1}{3}$ (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) $\frac{3}{4}$
59. Consider three curves
 $H : (x + a)y = \lambda; \lambda < 0, a > 0$
 $C : x^2 + y^2 - 21y + 109 = 0$
 $P : y^2 = bx; b > 0$
 Let the line $2x - y + 8 = 0$ touches the curve H, C and P at L, M & N respectively such that $LM = MN = \sqrt{45}$, then the value of $8\lambda + b + a$ is
- (A) 0 (B) 1
(C) -1 (D) 2
60. A line $L = 2x - y + 5 = 0$ is a tangent to the hyperbola $H \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that foot of perpendicular from the foci of hyperbola on line L is $\left(\frac{\sqrt{3} - 2\sqrt{5}}{\sqrt{5}}, \frac{2\sqrt{3} + \sqrt{5}}{\sqrt{5}} \right)$. If the hyperbola intersects an ellipse $E \equiv \frac{x^2}{25} + \frac{y^2}{\lambda^2} = 1$ orthogonally, then eccentricity of the ellipse is
- (A) $\sqrt{\frac{1}{5}}$ (B) $\sqrt{\frac{3}{5}}$
(C) $\frac{1}{5}$ (D) $\sqrt{\frac{2}{3}}$
61. Let $y = y(x)$ be a solution curve of the differential equation $\frac{dy}{dx} + \frac{y \cos x}{\sin x + \cos x} = \frac{2 \tan^2 x + \tan x + 2}{\sqrt{\sin x + \cos x}}$.
 If $y\left(\frac{\pi}{4}\right) = 2^{3/4}$, then $y^2\left(\frac{7\pi}{12}\right)$ is
- (A) $7\sqrt{6} + 4\sqrt{2}$ (B) $7\sqrt{2} + 4\sqrt{6}$
(C) $3\sqrt{6} + 7\sqrt{2}$ (D) $28\sqrt{2} + 16\sqrt{6}$
62. Let $P(z)$, $R(z^4)$ and $Q(z^2)$ be three points in Argand plane such that $PR + RQ = PQ$; ($z \neq 0, 1$) then z lies on
- (A) a circle centred at $\left(-\frac{1}{2}, 0\right)$ (B) a line segment having length $2\sqrt{3}$
(C) a line having slope $-\frac{1}{2}$ (D) a line parallel to y-axis
63. Let $\begin{vmatrix} a & \sqrt{5} & \sqrt{7} \\ \sqrt{3} & b & \sqrt{7} \\ \sqrt{3} & \sqrt{5} & c \end{vmatrix} = 0$, ($a \neq \sqrt{3}, b \neq \sqrt{5}, c \neq \sqrt{7}$) and $\frac{a}{a-\sqrt{3}} + \frac{b}{b-\sqrt{5}} + \frac{c}{c-\sqrt{7}} = \lambda$, if $a = 2\sqrt{3}$ then the point (b^2, c^2) may lie on the line
- (A) $5y - 21x = 0$ (B) $5y - 24x = 0$
(C) $5y - 63x = 0$ (D) $21y - 5x = 0$

64. In $\triangle ABC$, $\angle A = \tan^{-1}7$, $\angle C = \tan^{-1}\frac{4}{3}$. Let D be an interior point on side AC such that area of ABD is twice that of area of $\triangle BCD$. If $\angle ABD = \theta$, then the value of $\tan 2\theta$ is
- (A) $\frac{12}{5}$ (B) $\frac{5}{12}$
 (C) $\frac{36}{77}$ (D) $\frac{77}{36}$
65. There are two townships A and B in a city containing 40% and 60% of the population respectively. 15% of the total population suffer from heart disease.
 $P(\text{person suffering from heart disease who lives in A}) = 6 P(\text{person suffering from heart disease who lives in B})$ where $P(E)$ denotes the probability of a event E.
 A person randomly diagnosed turns out to be free from heart disease, then the probability that he lives in B township is
- (A) $\frac{28}{85}$ (B) $\frac{38}{85}$
 (C) $\frac{57}{85}$ (D) $\frac{47}{85}$
66. Consider the following frequency distribution
- | Class : | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 |
|-------------|--------|---------|---------|---------|---------|---------|
| Frequency : | 3 | a | b | 11 | 8 | 4 |
- If the mean is $\frac{277}{9}$ and median is $\frac{335}{11}$, then the value of $4a + b$ is
- (A) 40 (B) 38
 (C) 50 (D) 48
67. Consider the following statements
 P : Sachin is a topper
 Q : Sachin is sincere
 R : Sachin is not demotivated
 The negation of the statement.
 If Sachin is a sincere and he is not demotivated, then he is a topper
- (A) $P \rightarrow (Q \vee R)$ (B) $(\sim P) \wedge (Q \wedge R)$
 (C) $P \rightarrow ((\sim Q) \vee (\sim R))$ (D) $P \rightarrow (Q \wedge R)$
68. The upper four-fifth portion of a vertical tower subtends an angle $\tan^{-1}\frac{8}{21}$ at a point A in the horizontal plane through its foot and at a distance 50m from the foot. If the angle subtended by the lower one fifth of tower at point A is β , then the height of the tower can be
- (A) 25 (B) 50
 (C) 13 (D) 75
69. Let N be the number of four digits even numbers such that if 3 is one of the digit, then 5 is the succeeding digit. If N can be resolved as $P_1^\alpha P_2^\beta P_3^\gamma$ (P_1, P_2, P_3 are primes and α, β, γ are natural numbers then $P_1 + P_2 + P_3$ is
- (A) 11 (B) 31
 (C) 29 (D) 9

70. Let $S_1 = \{z \in \mathbb{C} : |z - 2| \leq |\operatorname{Re} z + 2|\}$
 $S_2 = \{z \in \mathbb{C} : z(1 + i) + \bar{z}(1 - i) - 12 \leq 0\}$
 $S_3 = \{z \in \mathbb{C} : \operatorname{Re} z \geq 0, \operatorname{Im} z \geq 0\}$
and $S = S_1 \cap S_2 \cap S_3$, then the maximum value of $|z - 20i|^2$; $z \in S$
(A) 436 (B) 446
(C) 456 (D) 384

SECTION - B

(Numerical Answer Type)

This section contains 5 Numerical based questions. The answer to each question is rounded off to the nearest integer value.

71. If the sum of values of θ in $(-3\pi, 3\pi)$ satisfying

$$\sum_{m=1}^{15} \sec\left(\theta + (m-1)\frac{\pi}{18}\right) \sec\left(\theta + m\frac{\pi}{18}\right) = (4 + 2\sqrt{3}) \operatorname{cosec} \frac{\pi}{18}$$
 is $\frac{k\pi}{10}$
then the value of k is _____
72. If $2(\sin A - \sin^3 A) = \cos B$
 $2(\cos A + \cos^3 A) = \sin B$; $0 < A, B < \pi/2$, then the value of $\cos B = \sqrt{\frac{m}{n}}$, m & n are co-prime
where $m + n$ is _____
73. Let $\frac{5}{6} \cos^{-1} \sqrt{\frac{3}{3+\pi^2}} + \frac{1}{3} \sin^{-1} \frac{2\sqrt{3}\pi}{3+\pi^2} + \frac{1}{6} \tan^{-1} \frac{\sqrt{3}}{\pi} = a$
and $\cos^{-1} \left[\frac{13}{40} \cos\left(\cot^{-1} \frac{5}{12}\right) + \frac{13}{32} \sin \cos^{-1} \left(\frac{5}{13}\right) \right] = b$
then $\operatorname{cosec} \left(\int_b^a \left[\frac{\tan x}{\sqrt{3}} \right] dx \right)$ is ([.] represents greatest integer function)
74. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ \sqrt{a} & 1 & 0 \\ a\sqrt{a} & \sqrt{b} & 1 \end{bmatrix}$; $a, b \in \mathbb{R}^+$. If for some $n \in \mathbb{N}$, $A^n = \begin{bmatrix} 1 & 0 & 0 \\ 72 & 1 & 0 \\ 3600 & 72 & 1 \end{bmatrix}$ then number of
triangles formed by joining the vertices of n sided polygon having no side common with the polygon are _____
75. Consider the curves
 $C_1 : y^2 - x = 0$
 $C_2 : y - x^2 = 0$; $0 \leq x \leq \frac{\sqrt{3}}{2}$
 $C_3 : y = f(x)$; $f(x) < 0 \quad \forall x \in \left(0, \frac{\sqrt{3}}{2}\right)$
From any point P on C_2 , lines are drawn parallel to the coordinate axes so as to intersect C_1 at Q and C_3 at R . If area of region $OPRO$ is twice of the area of region $OPQO$; (O being origin) then
the value of $\left| 32f\left(\frac{1}{2}\right) \right|$ is _____

SOLUTIONS

Physics

PART – A

SECTION – A

1.

B

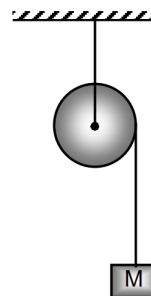
Sol. Applying conservation of angular momentum of system about centre of mass of disc

$$mvR = mv'R + \frac{2mR^2}{2}\omega$$

$$v' = R\omega$$

$$mvR = mv'R + mv'R$$

$$v' = \frac{v}{2} = \frac{\sqrt{2g\ell}}{2} = \sqrt{\frac{g\ell}{2}}$$



2.

D

Sol. $I = neA (\mu_e + \mu_h) E$

$$= 1.0 \times 10^6 \times 1.6 \times 10^{-19} \times 4 \times 10^{-4} (1.0) \times \frac{2}{2 \times 10^{-3}} = 6.4 \times 10^{-14} \text{ A}$$

3.

C

Sol. Basic concepts

4.

C

Sol. $C = C_V + \frac{dW}{ndT}$

$$\int dW = \int_{T_0}^{3T_0} \alpha T dT - \int_{T_0}^{3T_0} \frac{5R}{2} dT \text{ for one mole}$$

$$W = (4\alpha T_0 - 5R)T_0$$

5.

C

Sol. $\frac{dN}{dt} = R - \lambda N$

So $\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt \Rightarrow \ln(R - \lambda N) \Big|_0^N = -\lambda t$

So $\ln \frac{R - \lambda N}{R} = -\lambda t$

So $1 - \frac{\lambda N}{R} = e^{-\lambda t} \Rightarrow N = \frac{R}{\lambda} (1 - e^{-\lambda t})$

6.

C

Sol. Zener current will be maximum, when $V = 15V$

$$15 - (i \times 2.5k) = 5$$

$$i = 4 \text{ mA}$$

$$i_z = 3 \text{ mA}$$

7.

B

Sol. $\frac{R_{Ge}}{R_{Be}} = \left(\frac{X}{9}\right)^{1/3}$

$$X = 72$$

$$\text{Number of neutron} = 72 - 32 = 40$$

8.

B

Sol. frequency received by submarine

$$n_1 = \left(\frac{1400 + 100}{1400}\right) \times 40 \times 10^3 = \left(\frac{15}{14}\right) \times 40 \times 10^3$$

Frequency reflected by submarine

$$n_2 = \left(\frac{1400}{1400 - 100}\right) n_1 = \left(\frac{15}{13}\right) \times 40 \times 10^3 \text{ Hz} = \frac{600}{13} \text{ kHz}$$

9.

C

Sol. $f = \frac{2\pi(1 - \cos \theta_C)}{4\pi}$

$$f = \frac{2\pi}{4\pi} \left[1 - \sqrt{1 - \frac{1}{\mu^2}} \right] = \frac{1}{2} \left[1 - \sqrt{1 - \frac{1}{\mu^2}} \right]$$

10.

D

Sol. For $(r \leq R)$

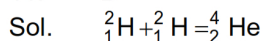
$$E \times 4\pi r^2 = \int_0^r cx 4\pi x^2 dx / \epsilon_0$$

$$\text{So } E \propto r^2$$

and For $(r \geq R)$

$$E \propto \frac{1}{r^2}$$

11. D



$$\text{BE}_\text{H} = 1.1 \text{ MeV}$$

$$\text{BE}_\text{He} = 7 \text{ MeV}$$

$Q_\text{value} = \text{Binding energy of product} - \text{Binding energy of reactants}$

$$= (4 \times 7 - 4 \times 1.1) \text{ MeV} = 23.6 \text{ MeV}$$

$$= 23.6 \times 1.6 \times 10^{-19} \times 10^6 = 37.76 \times 10^{-23}$$

12. C

Sol. $B = \frac{\mu_0 dI}{2\pi R} = \frac{\mu_0 Id_0}{(2\pi R)(2\pi R)} = \frac{\mu_0 Id_0}{4\pi^2 R^2}$

13. D

Sol. After long time,

$$I_A = \frac{2V}{7R} = 20 \text{ mA}$$

...(i)

Immediately after the key is closed,

$$I = \frac{V}{3R} = \frac{70}{3} \text{ mA}$$

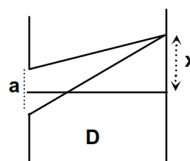
14. C

Sol. For 1st secondary minima $\frac{ax}{D} = \lambda_1$

For 1st secondary maxima $\frac{ax}{D} = \frac{3\lambda_2}{2}$

$$\lambda_1 = \frac{3}{2}\lambda_2$$

$$\lambda = 440 \text{ \AA}$$

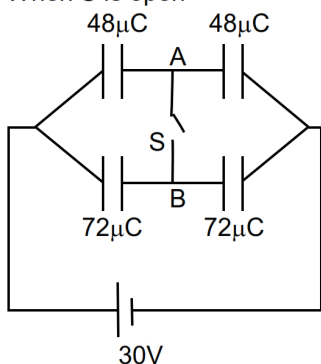


15. D

Sol. Basic Concepts

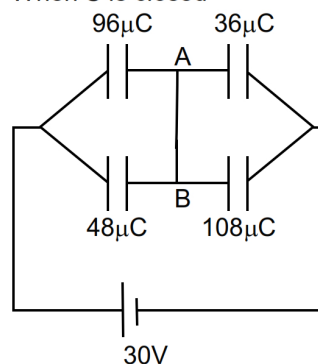
16. A

Sol. When S is open



So charge flown = $60 \mu\text{C}$
From A to B

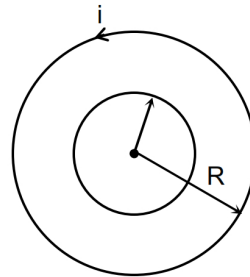
When S is closed



17. B
Sol. Basic concept

18. C
Sol. $E2\pi r = \frac{d}{dt}(B\pi r^2)$
 $E = \frac{r}{2} \frac{dB}{dt}$
 $\tau = QEr = \frac{Qr^2}{2} \frac{dB}{dt}$

19. B
Sol. $B = \frac{\mu_0 i}{2R}$
 $\phi = B\pi r^2 = \frac{\mu_0 i \pi r^2}{2R}$
 Mutual inductance, $M = \frac{\phi}{i} = \frac{\mu_0 \pi r^2}{2R}$



20. D
Sol. Rate of cooling
 $-\frac{dT}{dt} = \frac{\epsilon \sigma A(T^4 - T_s^4)}{mC} = r$
 $\frac{r_{\text{sphere}}}{r_{\text{cube}}} = \frac{m_{\text{cube}}}{m_{\text{sphere}}} = \frac{\rho a^3}{\rho \frac{4}{3} \pi r^3}$
 given, $4\pi r^2 = 6a^2$
 $\frac{r_{\text{sphere}}}{r_{\text{cube}}} = \frac{3 \left(\frac{4\pi}{6} \right)^{3/2} r^3}{4\pi r^3} = \sqrt{\frac{\pi}{6}}$

SECTION – B

21. 6
Sol. Separation between two maxima $\beta = \frac{\lambda D}{d}$

22. 40
Sol. $v_0^2 = 4v_0^2 - (2)(\mu g)(d) \Rightarrow 2\mu g d = 3v_0^2$
 $v_0^2 = v^2 - (2)(5\mu g)(d) \Rightarrow v^2 = v_0^2 + 15V_0^2$
 So $v = 4v_0$

23. 1
Sol. $\frac{2v}{2(\ell_0 + 1.2d)} = \frac{3v}{4(\ell_c + e)}$

$$\text{So } 4(23 + e) = 3(30 + 2e)$$

$$\text{So } e = 1 \text{ cm}$$

24. 9

$$\text{Sol. } E_0 = CB_0$$

$$= 3 \times 10^8 \times 3 \times 10^{-8} = 9 \text{ V/m}$$

25. 30

$$\text{Sol. } \frac{1}{f_e} = -2\frac{1}{f_\ell} + \frac{1}{f_m},$$

$$\text{Where } \frac{1}{f_\ell} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{40}\right) = \frac{1}{120}$$

$$\text{So } \frac{1}{f_e} = -2\frac{1}{120} - \frac{1}{20} = -\frac{1}{15} \text{ cm}$$

$$\text{So } R = 30 \text{ cm}$$

Chemistry

PART – B

SECTION – A

26. B

Sol. $P_1 V_1 = P_2 V_2$

$$10\text{m} \times (4) \times A = (10 + h)\text{m} \times h \times A$$

$$h = 3.06 \text{ m}$$

27. C

Sol. Dithionous acid $\rightarrow \text{H}_2\text{S}_2\text{O}_4$

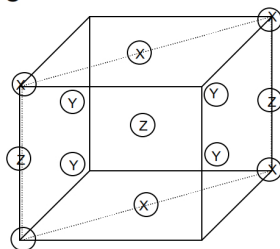
$$S = +3$$

Dithionic acid $\rightarrow \text{H}_2\text{S}_2\text{O}_6$

$$S = +5$$

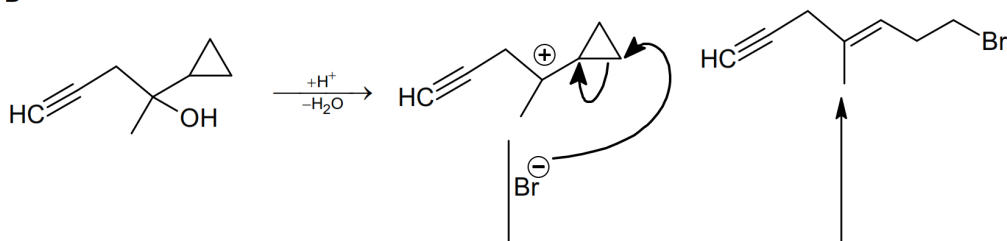
28. C

Sol.



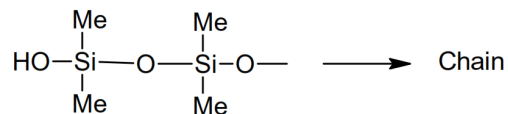
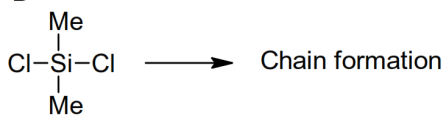
29. D

Sol.



30. B

Sol.



31. C

Sol. LiClO_4 is more soluble than LiF .

32. D

Sol. Factual

33. B

Sol. BF_3 is electron deficient and is used to generate electrophile in F.C. reactions. Therefore, acts as catalyst in industrial processes.

34. B
Sol. Uridine is glycosylated pyrimidine-containing uracil attached to ribose ring.
Ribose(sugar) + Uracil(base) $\longrightarrow$ Nucleoside

35. A
Sol. Degenerate orbitals = n^2
 $= (5)^2 = 25$

36. D
Sol. $\Delta S \rightarrow$ State function $\Delta S = \Delta S_1 + \Delta S_2$
 $\Delta S = \frac{q}{T}$ since q = same; smaller temperature, higher entropy.

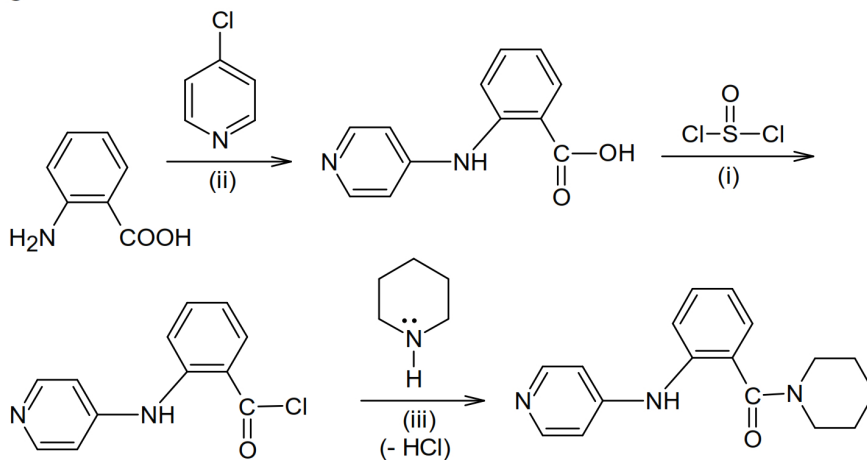
37. D
Sol. $Ti^{+3} \longrightarrow 3d^1 \longrightarrow t_{2g}^1$
 $Cr^{+3} \longrightarrow 3d^3 \longrightarrow t_{2g}^3$
 $Mn^{+3} \longrightarrow 3d^4 \longrightarrow t_{2g}^3 eg^1$
 $Co^{+3} \longrightarrow 3d^6 \longrightarrow t_{2g}^4 eg^2$

38. A
Sol. Factual

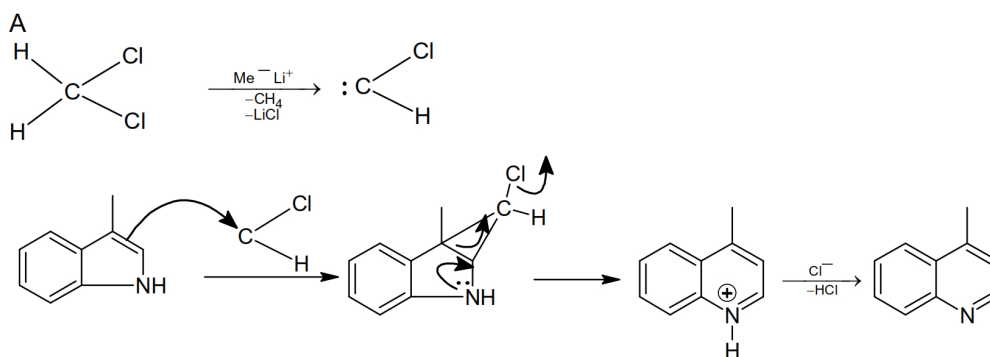
39. B
Sol. MnO_2 oxidises unhindered primary allylic and benzylic alcohols to aldehyde.

40. C
Sol. $\frac{1}{x_o - a} - \frac{1}{x_o} = k_2 t$
Rearranging
 $\frac{a}{x_o - a} = x_o k_2 t$
 $\therefore \frac{a}{x_o - a}$ vs. t gives $x_o k_2$ as slope.

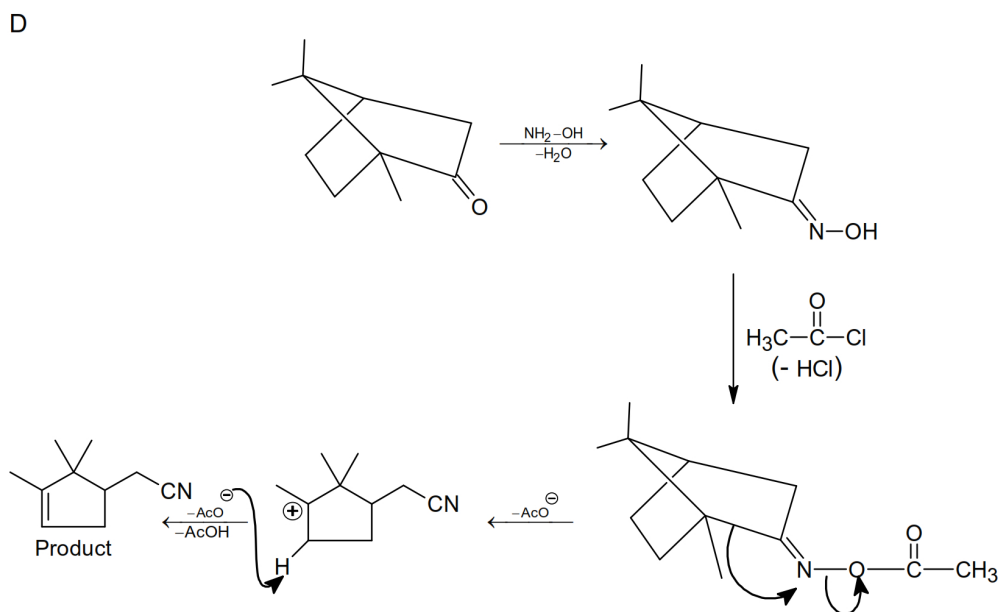
41. C
Sol.



42.
Sol.



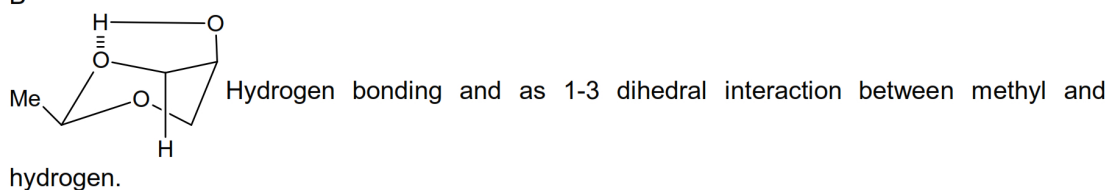
43.
Sol.



44.

B

Sol.



45.

B

Sol.

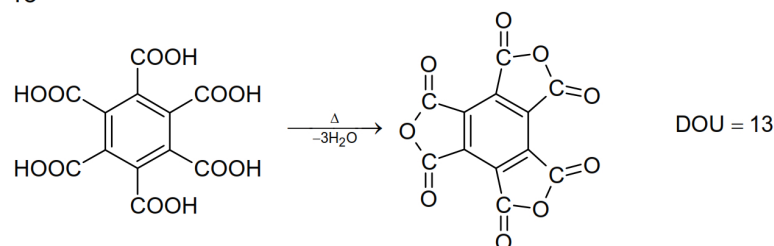
$$x_A = \frac{0.744 + 0.359Z_{\text{eff}}}{r_A^2} \leftarrow \text{Alfred - Rochow scale}$$

SECTION - B

46.

13

Sol.



47. 2

Sol. $[H^+] = 10^{-4}$

$$\therefore [Ca^{+2}] = \frac{10^{-4}}{2} M$$

Weight of $Ca^{+2} = 20 \times 10^{-4}$ g in 1 L

$$\therefore \text{Weight of } Ca^{+2} \text{ ion in } 10^6 \text{ mL } H_2O = \frac{20 \times 10^{-4} \times 10^6}{10^3} = 2$$

$\therefore$ Hardness = 2 ppm.

48. 1

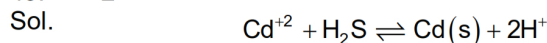
Sol. Theoretical efficiency = $\frac{-\Delta G^\circ}{-\Delta H^\circ}$

$$0.84 = \frac{-\Delta G^\circ}{285}$$

$$\Delta G^\circ = -0.84 \times 285 \text{ kJ} = -nFE_{\text{cell}}^\circ$$

$$E_{\text{cell}}^\circ = 1.24 \text{ V}$$

49. 2



Initial 0.1×10^{-3} moles

$0.08 \times 10 \times 10^{-3}$ moles

final

—

$0.8 \times 10^{-3} + 0.2 \times 10^{-3}$

$= 10^{-3}$ moles

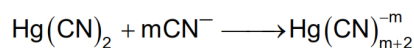
$$\therefore [H^+]_{\text{final}} = \frac{10^{-3}}{100} \times 1000 = 0.01 \text{ M} = 10^{-2} \text{ M}$$

pH = 2.0

50. 8

Sol. For KCN: $\Delta T_f = i \cdot K_f \cdot m \Rightarrow 0.8 = K_f \cdot 0.2 \times 2$

$$K_f = 2 \quad \dots (i)$$



Initial 0.1 0.2 —

Final $(0.2 - 0.1 m)$ 0.1

$$\text{Final effective molality} = (0.2 - 0.1 m) + 0.1 + 0.2 \\ = 0.5 - 0.1 m$$

$$\text{New: } \Delta T_f = K_f \cdot m$$

$$0.6 = 2 \times (0.5 - 0.1 m)$$

$$m = 2$$

$$4m = 8$$



SECTION – A

51. B

Sol. $A = \{1, 2, 3, \dots, 141\}$

$$A \cap (B - C) = \{5, 11, 14, 17, 23, 26, 29, \dots, 134, 137\}$$

$$\text{Sum of elements} = (5 + 11 + 17 + \dots + 137) + (14 + 26 + 38 + \dots + 134)$$

52. C

Sol. $x^2 - ax + 4 = 0$ must have atleast one positive root

$$\therefore \alpha\beta > 0$$

$\therefore$ both roots must be positive

$$\Rightarrow a \geq 4$$

53. B

$$\text{Sol. } 30(1 + x + x^2)^{30} (1 + 2x) = \sum_{r=0}^{60} r a_r x^{r-1} (1 + x + x^2)$$

Equating coefficient of x^{20} on both sides we have,

$$30(a_{20} + 2a_{19}) = 21a_{21} + 20a_{20} + 19a_{19}$$

$$\Rightarrow 21a_{21} = 10a_{20} + 41a_{19}$$

54. C

Sol. Use $\int (xg'(x) + g(x))dx = xg(x) + C_1$

55. C

$$\text{Sol. } k = 2500 = 2^2 \cdot 5^4$$

$\therefore$ number of factor = 5

56. B

$$\text{Sol. } b = -4, c = 9$$

$$2\sin^2\theta = 3\cos\theta$$

$$\Rightarrow 2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$\cos\theta = \frac{1}{2} \therefore \sin^2\theta = \frac{3}{4}$$

$$\therefore 4x^2 - 4y^2 + xy + 12x + \frac{3}{2}y + 9 = 0$$

57. A

$$\text{Sol. } r = \frac{r_1 r_2}{r_1 + r_2 + 2\sqrt{r_1 r_2}}$$

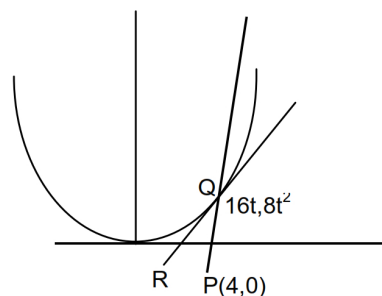
58. B

$$\text{Sol. } A = \frac{1}{2} (4 - 8t)8t^2 = 16(t^2 - 2t^3)$$

$$\frac{dA}{dt} = 16(2t - 6t^2) = 0 \Rightarrow t = 0, \frac{1}{3}$$

$$\frac{d^2A}{dt^2} = 16(2 - 12t) < 0 \text{ at } t = \frac{1}{3} \Rightarrow A \text{ is maximum}$$

$$\therefore Q\left(\frac{16}{3}, \frac{8}{9}\right) \text{ lies on the line } \therefore m = \frac{2}{3}$$



59.

A

Sol.

M is (1, 10)

 $\therefore$ N is (4, 16)

L is (-2, 4)

Equation of tangent to $(x + a)y = \lambda$ is

$$y = 2x + \frac{\lambda}{t} + 2(a - t); \lambda = -2t^2$$

$$\therefore 2a - 4t = +8 \Rightarrow a - 2t = 4$$

$$\text{and } (a - 2)4 = \lambda = -2t^2$$

$$\therefore (t + 2)^2 = 0$$

$$\Rightarrow t = -2$$

$$\therefore \lambda = -8 \text{ \& } a = 0$$

60.

B

Sol.

$$4a^2 - b^2 = 25$$

Equation of auxiliary circle is $x^2 + y^2 = 8$

$$\Rightarrow a^2 = 8, b^2 = 7$$

$$\text{Also } a^2 + b^2 = A^2 - \lambda^2$$

$$\Rightarrow \lambda^2 = 25 - 15 = 10$$

$$\therefore \text{eccentricity of ellipse } e = \sqrt{1 - \frac{10}{25}} = \sqrt{\frac{3}{5}}$$

61.

D

Sol.

$$IF = e^{x/2} \sqrt{\sin x + \cos x}$$

$$\therefore \text{solution is } ye^{x/2} \sqrt{\sin x + \cos x} = \int e^{x/2} (\tan x + 2 \sec^2 x) dx$$

$$ye^{x/2} \sqrt{\sin x + \cos x} = 2e^{x/2} \tan x + c$$

$$\therefore y\left(\frac{\pi}{4}\right) = 2^{3/4} \therefore c = 0$$

$$y = \frac{2 \tan x}{\sqrt{\sin x + \cos x}}$$

62.

D

Sol.

$$\text{Arg}\left(\frac{z^4 - z}{z^2 - z}\right) = 0$$

$$\Rightarrow \text{Arg}\left(\frac{z(z-1)(z^2+z+1)}{z(z-1)}\right) = 0; z \neq 0, 1$$

$$\Rightarrow z^2 + z + 1 \text{ is positive real number } (x + iy)^2 + x + iy + 1 > 0$$

$$\Rightarrow x^2 - y^2 + x + 1 > 0 \text{ and } y(1 + 2x) = 0$$

$$(i) \quad y = 0, x^2 + x + 1 > 0 \Rightarrow x \in \mathbb{R}$$

$$(ii) \quad x = -\frac{1}{2}, y^2 < \frac{3}{4} \Rightarrow y \in \left(-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right).$$

63.

C

Sol.

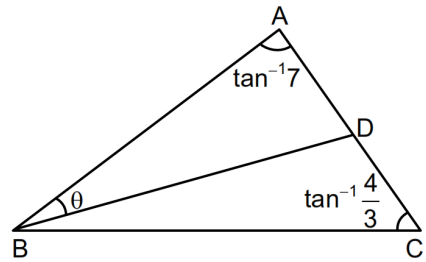
$$\frac{a}{a-\sqrt{3}} + \frac{b}{b-\sqrt{5}} + \frac{c}{c-\sqrt{7}} = 2$$

$$\text{If } a = 2\sqrt{3}, \text{ then } \frac{b}{b-\sqrt{5}} + \frac{c}{c-\sqrt{7}} = 0$$

$$\text{Possible value of } b = \frac{2\sqrt{5}}{3}, c = 2\sqrt{7}.$$

64. D

$$\begin{aligned} \text{Sol. } \frac{\frac{1}{2} \sin\left(\tan^{-1} \frac{4}{3}\right) BD \sin \theta}{\frac{1}{2} \sin\left(\tan^{-1} 7\right) BD \sin\left(\frac{\pi}{4} - \theta\right)} &= 2 \\ \Rightarrow \frac{\frac{4}{5} \sin \theta}{\frac{7}{5\sqrt{2}} \left(\frac{\cos \theta - \sin \theta}{\sqrt{2}}\right)} &= 2 \Rightarrow \cot \theta = \frac{11}{7} \end{aligned}$$



65. C

$$\begin{aligned} \text{Sol. } P(A) &= \frac{2}{5}, P(B) = \frac{3}{5} \\ P(D) &= \frac{3}{20} \\ 6 \times \frac{2}{5} x + \frac{3}{5} \times x &= \frac{3}{20} \Rightarrow x = \frac{1}{20} \\ P\left(\frac{B}{D^c}\right) &= \frac{P(B \cap D^c)}{P(D^c)} = \frac{\frac{3}{5} \times \frac{19}{20}}{\frac{85}{100}} = \frac{57}{85} \end{aligned}$$

66. A

$$\text{Sol. } \frac{\sum_{i=1}^n x_i f_i}{\sum_{i=1}^n f_i} = \frac{15a + 25b + 980}{26 + a + b} = \frac{277}{9} \quad \dots(1)$$

$$\text{Also } 30 + \frac{26 + a + b}{2} - (a + b + 3) \times 10 = \frac{335}{11}$$

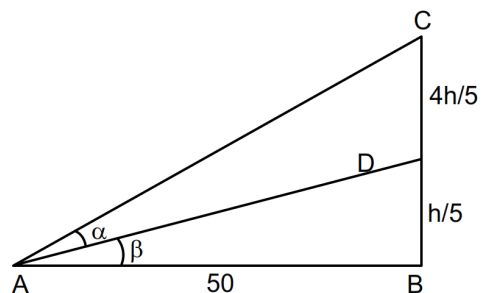
$$\Rightarrow a + b = 19 \quad \dots(2)$$

From (1) and (2) $a = 7$, $b = 12$.

67. B

68. A

$$\begin{aligned} \text{Sol. } \tan(\alpha + \beta) &= \frac{h}{50} \\ \Rightarrow \frac{h}{50} &= \frac{\frac{8}{21} + \frac{h}{250}}{1 - \frac{8}{21} \times \frac{h}{250}} = \frac{2000 + 21h}{5250 - 8h} \\ h &= 25 \end{aligned}$$



69.

B

Sol.

$$\boxed{3} \boxed{5} \dots = 9 \times 5 = 45$$

$$\boxed{3} \boxed{5} \dots = 8 \times 5 = 40$$

$$\dots = 8 \times 9 \times 9 \times 5 = 3240$$

$$N = 3325$$

70.

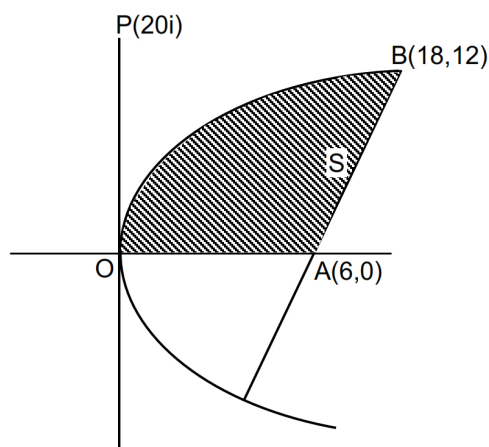
A

Sol.

$$PB = \sqrt{324 + 64} = \sqrt{388}$$

$$PA = \sqrt{36 + 400} = \sqrt{436}$$

$$OP = 20$$



SECTION - B

71.

5

Sol.

$$\sum_{m=1}^{15} \frac{\sin\left(\theta + m\frac{\pi}{18} - \left(\theta + (m-1)\frac{\pi}{18}\right)\right)}{\cos\left(\theta + m\frac{\pi}{18}\right)\cos\left(\theta + (m-1)\frac{\pi}{18}\right)}$$

$$= \sum_{m=1}^{15} \tan\left(\theta + m\frac{\pi}{18}\right) - \tan\left(\theta + (m-1)\frac{\pi}{18}\right)$$

$$\Rightarrow \tan\left(\theta + \frac{5\pi}{6}\right) - \tan\theta = 4 + 2\sqrt{3}$$

$$\Rightarrow \cos\left(2\theta + \frac{5\pi}{6}\right) = 1$$

$$\Rightarrow \theta = n\pi - \frac{5\pi}{12}; n \in \mathbb{I}.$$

72.

59

Sol.

Squaring and adding

$$1 + \sin^6 A + \cos^6 A + 2(\cos^4 A - \sin^4 A) = 1/4$$

$$\Rightarrow 2 + 1 - 3\sin^2 A \cos^2 A + 2\cos 2A = 1/4$$

$$2 - 3/4 \sin^2 2A + 2\cos 2A = 1/4$$

$$\Rightarrow 3\cos^2 2A + 8\cos 2A + 4 = 0$$

$$\cos 2A = -2/3$$

$$\sin A = \frac{\sqrt{5}}{6}$$

$$\therefore \cos B = 2\left(\sqrt{\frac{5}{6}} - \frac{5}{6}\sqrt{\frac{5}{6}}\right) = \sqrt{\frac{5}{54}}.$$

73. 13

Sol. $a = \frac{5\pi}{12}, b = \frac{\pi}{3}$

$$\int_{\pi/3}^{5\pi/12} \left[\frac{\tan x}{\sqrt{3}} \right] dx$$

$$= \int_{\pi/3}^{\tan^{-1} 2\sqrt{3}} 1 + \int_{\tan^{-1} 2\sqrt{3}}^{5\pi/12} 2 dx = \cot^{-1} 2\sqrt{3} = \operatorname{cosec}^{-1} 13.$$

74. 546

Sol. Let $A = I + B$ where $B = \begin{bmatrix} 0 & 0 & 0 \\ \sqrt{a} & 0 & 0 \\ a\sqrt{a} & \sqrt{b} & 0 \end{bmatrix}$, $B^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{ab} & 0 & 0 \end{bmatrix}$, $B^3 = 0$ (Null)

$$\therefore A^n = I + {}^nC_1 B + {}^nC_2 B^2$$

$$\Rightarrow A^n = \begin{bmatrix} 1 & 0 & 0 \\ n\sqrt{a} & 1 & 0 \\ na\sqrt{a} + \frac{n(n-1)}{2}\sqrt{ab} & n\sqrt{b} & 1 \end{bmatrix}$$

$$\Rightarrow n\sqrt{a} = n\sqrt{b} = 72 \Rightarrow a = b \text{ and } 72a + 36(72 - \sqrt{a}) = 3600$$

$$\Rightarrow \sqrt{a} = 4 \Rightarrow a = 16 \text{ and } n = 18.$$

75. 20

Sol. Let $P(t, t^2)$, $Q(t^4, t^2)$

$$\int_0^t (x^2 - f(x)) dx = 2 \int_0^{t^2} (\sqrt{y} - y^2) dy$$

$$\Rightarrow t^2 - f(t) = 2(t - t^4)2t$$

$$\Rightarrow f(t) = 4t^5 - 3t^2$$

$$f\left(\frac{1}{2}\right) = \frac{4}{32} - \frac{3}{4} = -\frac{20}{32}.$$

